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REGULAR REFLECTION OF A SHOCK WAVE FROM A WALL WITH A BEND

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This article examines the problem of disturbance of the nonsteady regular reflection of a shock wave. A disturbance is introduced into the main flow when the configuration comprised of the incident and reflected shock waves reach a bend in the wall. The problem is solved within the class of self-similar solutions. In a linear formulation, the perturbation problem reduces to a contact problem involving analytic functions. It is shown that the perturbation problem can be solved if the reflected shock is weak and very stable. The follows from the unchanging character of the flow. An analytic solution in integral form is obtained in this case. The contact problem cannot be solved if the reflected shock is strong, which indicates that the flow has undergone a fundamental restructuring.

1. Formulation of the Problem. Well will study a nonsteady gas flow created with the reflection of a shock wave from a wall with a band (Fig. 1). We introduce the cartesian coordinate system (X, Y), the center of which lies at the point of inflection. Let us suppose that at $t < 0$ a configuration comprised of the incident and reflected shock waves moves along the wall ($Y = 0$) through the quiescent gas at a velocity U. The main parameters of the flow in the regions ahead of and behind the incident shock front and behind the reflected front will be designated by the subscripts 0, 1, and 2, respectively. Let the incident shock front Γ_1 be given by the equation

$$X + bY + Ut = 0 \ (b = \text{const},\ U = \text{const},\ b > 0,\ U > 0),$$

and let the reflected from Γ_2 be given by the equation

$$X - aY + Ut = 0 \ (a = \text{const},\ a > 0).$$

The gas flow is steady in the coordinate system connected with the point of intersection of the fronts N. The parameters of the flow in regions 0, 1, and 2 are connected by the relations for an oblique shock wave:

$$\tau_i(\mathbf{W}_{i-1}\cdot\mathbf{n}_i) = \tau_{i-1}(\mathbf{W}_i\cdot\mathbf{n}_i),$$
$$P_{i-1} + \rho_{i-1}(\mathbf{W}_{i-1}\cdot\mathbf{n}_i)^2 = P_i + \rho_i(\mathbf{W}_i\cdot\mathbf{n}_i)^2,$$
$$\mathbf{W}_{i-1} - (\mathbf{W}_{i-1}\cdot\mathbf{n}_i)\mathbf{n}_i = \mathbf{W}_i - (\mathbf{W}_i\cdot\mathbf{n}_i)\mathbf{n}_i,$$
$$h_{i-1} + \frac{1}{2}(\mathbf{W}_{i-1}\cdot\mathbf{n}_i)^2 = h_i + \frac{1}{2}(\mathbf{W}_i\cdot\mathbf{n}_i)^2 \quad (i = 1, 2).$$

Here, P_j is pressure; ρ_i is density; $\tau_i = \rho_i^{-1}$; h_i is enthalpy; $\mathbf{n}_i$ is a normal to the front Γ_i; $\mathbf{W}_i$ is a vector connected with the velocity vector of the gas $\mathbf{V}_i = (u_i v_i)$ in regions 0, 1, 2 by the relation $\mathbf{W}_i = \mathbf{V}_i + (U, 0)$. By virtue of the impermeability condition, $v_2 = 0$ on the wall $Y = 0$.

At the time $t = 0$ let point N reach the coordinate origin, where the rigid wall is bent through the small angle $\varepsilon(\varepsilon > 0)$. The front of the reflected wave curves at $t > 0$ and one of two regions of disturbed flow is formed. In case (a), when $u_2 + c_2 \geq 0$, the region is bounded by the rigid wall, a section of the diffracted front of the reflected wave Γ_2, and the characteristic surface Γ_3 moving at the speed of sound:

$$(X - u_2 t)^2 + Y^2 = c_2^2 t^2$$

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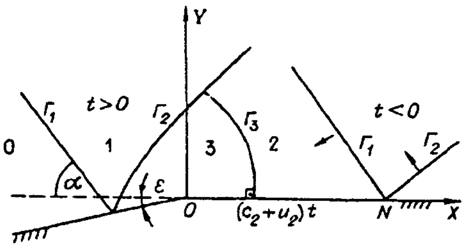

Fig. 1

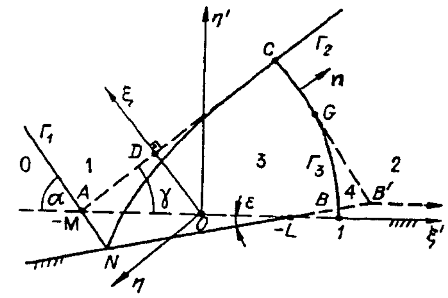

Fig. 2

($u_2 < 0$ is the horizontal component of velocity; c_2 is the speed of sound in region 2). In case (b), when $u_2 + c_2 < 0$, the region of disturbed flow is bounded by a segment of the rectilinear acoustic characteristic B'G, part of the acoustic circle GC, a section of the diffracted from Γ_2, and the rigid wall (Fig. 2). The characteristic B'G originates from the point of inflection and contacts the acoustic circle Γ_3 at point G. Case b is depicted by the dashed lines in Fig. 2.

Description of the flow at $t > 0$ reduces to solution of the equations of gas dynamics

$$\frac{d}{dt}\rho + \rho \operatorname{div} \mathbf{V} = 0, \quad \frac{d}{dt}\mathbf{V} + \frac{1}{\rho}\nabla P = 0, \qquad \frac{d}{dt}S = 0, \quad \rho = \rho(P,S) \quad \left(c^2 = \frac{dP}{d\rho}\right) \tag{1.1}$$

in the region of disturbed flow if the impermeability condition is satisfied on the wall and the Hugoniot condition is satisfied on the unknown boundary NC. Here, P is pressure; ρ is density; V is velocity; S is entropy; c is the speed of sound;

$$\frac{d}{dt} \doteq \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla.$$

Considering that $\varepsilon << 1$, we linearize the problem to obtain a piecewise-constant solution for P_2, ρ_2, V_2, and S_2 that describes the gas flow when $\varepsilon = 0$. We represent the flow parameters in the form

$$\begin{gathered} P = P_2 + \varepsilon\rho_2 c_2^2 p_j, \quad S = S_2(1 + \varepsilon s_j), \\ \mathbf{V} = \mathbf{V}_2 + \varepsilon c_2(u_j, v_j), \quad \rho = \rho_2(1 + \varepsilon\rho_j), \end{gathered} \tag{1.2}$$

where p_j, ρ_j, (u_j, v_j), s_j are dimensionless perturbations of the flow parameters. In case (b), the region in which the linear perturbation problem is solved breaks down into two subregions: subregion 3, the right boundary of which is the acoustic circle Γ_3; subregion 4, bounded by the acoustic circle Γ_3, the rigid wall, and the segment B'G (Fig. 2). In a linear formulation, the segment B'G can be regarded as a first-order discontinuity modeling the change in the flow in a wave centralized at the point of inflection. Accordingly, $j = 3, 4$ in (1.2). In case (a), $j = 3$. The given problem is self-similar, making it is convenient to introduce the following independent variables:

$$\xi = \frac{\alpha\eta' - \xi'}{\sqrt{\alpha^2 + 1}}, \quad \eta = -\frac{\alpha\xi' + \eta'}{\sqrt{\alpha^2 + 1}} \quad \left(\xi' = \frac{X - u_2 t}{c_2 t}, \quad \eta' = \frac{Y}{c_2 t}\right). \tag{1.3}$$

The solution is constant in region 4 and the quantities p_4, (u_4, v_4), ρ_4 and s_4 are determined from the relations for a first-order discontinuity. To determine the perturbations p_3, (u_3, v_3), p_3 in region 3, we need to solve the following system of equations (the 3 subscript will henceforth be omitted)

$$\xi\frac{\partial p}{\partial \xi} + \eta\frac{\partial p}{\partial \eta} = \frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta}, \quad \xi\frac{\partial u}{\partial \xi} + \eta\frac{\partial u}{\partial \eta} = \frac{\partial p}{\partial \xi}, \quad \xi\frac{\partial v}{\partial \xi} + \eta\frac{\partial v}{\partial \eta} = \frac{\partial p}{\partial \eta}, \quad \xi\frac{\partial s}{\partial \xi} + \eta\frac{\partial s}{\partial \eta} = 0, \tag{1.4}$$

which is obtained from system (1.1) after linearization and a changeover to new self-similar variables (1.3).

Since the last equation of the system can be integrated independently using the boundary conditions on the shock from and the acoustic circle, we will subsequently focus our attention on a system consisting of the first three equations.

Let us formulate the boundary conditions of the linear problem. The equation of the straight part of the front of the reflected shock wave takes the form

$$\xi = \frac{u_2 + U}{c_2}\frac{1}{\sqrt{a^2+1}} = \frac{M}{\sqrt{a^2+1}} = m.$$

We will study regimes of gas motion in which the Mach number M < 1 (the case M > 1 was described in detail in [1]). The equation of the disturbed part of the front of the reflected shock wave will be represented as $\xi = m + \varepsilon\Psi\eta)$. The linearized Hugoniot relations give the following boundary conditions at $\xi = m$:

$$p = \frac{2M(1 - R^{-1})}{\sqrt{1 + a^2}(\Delta_2 + 1)}(\Psi(\eta) - \eta\Psi_\eta), \quad v = \frac{(1 - R)M}{\sqrt{1 + a^2}}\Psi_\eta, \quad u = \frac{(1 - \Delta_2)\sqrt{1 + a^2}}{2M}p. \tag{1.5}$$

After removing $\Psi(\eta)$ and $\Psi_\eta(\eta)$ from (1.5) we obtain

$$u = Ap, \quad \eta\frac{\partial v}{\partial\eta} = B\frac{\partial p}{\partial\eta}, \tag{1.6}$$

where

$$A = \frac{(1 - \Delta_2)}{2m}; \; B = \frac{(\Delta_2 + 1)R}{2}; \; \Delta_2 = \frac{P_2 - P_1}{\tau_1 - \tau_2}\left(\frac{\partial\tau}{\partial P}\right)_2; \; R = \frac{\rho_2}{\rho_1}.$$

Relations (1.6) are satisfied on the straight line ξ = m (see Fig. 2). The front of weak disturbances originating from the vertex of the angle is described by the equation $\xi^2 + \eta^2 = 1$.

Thus, the mathematical formulation of the problem is such as to require solution of Eqs. (1.4) with the following boundary conditions: on the boundary $\eta' = 0$

$$-L = \frac{-u_2}{c_2} \leq 1, \; \begin{cases} v' = M & \text{when } \xi' < -L, \\ v' = 0 & \text{when } \xi' > -L, \end{cases}$$

$$-L = \frac{-u_2}{c_2} > 1, \; v' = M$$

(v' is the projection of the velocity vector on the axis $O\eta'$); Eqs. (1.6) are valid on the boundary ξ = m in both cases, u = v = p = 0 on the acoustic characteristic Γ_3 in case (a), and

in case (b)

$$\begin{cases} u = v = p = 0 \text{ (on the arc } CG), \\ p = \dfrac{ML}{\sqrt{L^2 - 1}} \; (u, v - \text{(u, v are assigned constants which are nontrivial on the arc GB)}) \end{cases}$$

2. Boundary-Value Problem for Pressure. We obtain the following equation for p after excluding u and v from Eqs. (1.4)

$$\left(\xi\frac{\partial}{\partial\xi} + \eta\frac{\partial}{\partial\eta} + 1\right)\left(\xi\frac{\partial p}{\partial\xi} + \eta\frac{\partial p}{\partial\eta}\right) = \nabla^2 p, \tag{2.1}$$

which will be hyperbolic at $\xi^2 + \eta^2 > 1$ and elliptic at $\xi^2 + \eta^2 < 1$. By virtue of Eqs. (1.4), the condition on the axis η' = 0 is written in the form

$$\frac{\partial p}{\partial\eta'} = ML\delta_0(\xi' + L) \qquad \text{in case (a)}$$

$$\frac{\partial p}{\partial\eta'} = 0 \qquad \text{in case (b)}$$

($\delta_0(\xi)$ is the delta function). The below condition follows from Eqs. (1.4) and (1.6) for the section ξ = m

$$m\left(m\frac{\partial p}{\partial\xi} + \eta\frac{\partial p}{\partial\eta}\right) = \frac{\partial p}{\partial\xi} - \left(A\eta - \frac{Bm}{\eta}\right)\frac{\partial p}{\partial\eta}.$$

On the boundary between the regions of uniform and disturbed flow $\xi^2 + \eta^2 = 1$ we have

$$\frac{\partial p}{\partial s} = 0 \qquad \text{in case (a),}$$

$$\frac{\partial p}{\partial s} = \frac{ML}{\sqrt{L^2 - 1}}\delta_0(\theta - \theta_G) \qquad \text{in case (b).}$$

Here, s is a tangent vector to Γ_3 directed counterclockwise; θ is a polar angle in the plane (ξ, η). Along the line AC, integral conditions describing the change in velocity v along AC from v_N to zero must be satisfied

$$\int_{AC} \frac{B}{\eta}\, dp = -v_N, \tag{2.2}$$

as well as integral conditions describing the change in pressure

$$\int_{AC} dp = \frac{P_2 - P_N}{\rho_2 c_2^2} = -p_N \tag{2.3}$$

(P_N is pressure at the point N). The value of P_N is calculated from the theory of regular reflection [1, 2]. We perform this calculation using Eqs. (1.5), the condition at point A $au - v = M\sqrt{1 + a^2}$, and the condition for passage of the reflected front through point N (see Fig. 2), Thus

$$p_N = \frac{2m(1 - R)}{a^2(1 - \Delta_2) - R(1 + \Delta_2)}\left(\frac{U}{\sqrt{1 + a^2}}(a + \operatorname{ctg}\alpha) + \frac{am(1 + a^2)}{1 - R}\right),$$

Then we write v_N from the second relation of Eqs. (1.5) as

$$v_N = \frac{a(1 - \Delta_2)p_N - 2m^2(1 + a^2)}{2m}$$

(α is the angle of incidence of the front Γ_1).

3. Transformation of the Linearized Problem. By resorting to the successive transformations

$$\xi = r\cos\theta,\ \eta = r\sin\theta,\ r_1 = \frac{1 - \sqrt{1 - r^2}}{r},\ \theta_1 = \theta$$

we reduce Eq. (2.1) to the Laplace equation

$$\frac{\partial^2 p}{\partial r_1^2} + \frac{1}{r_1}\frac{\partial p}{\partial r_1} + \frac{1}{r_1^2}\frac{\partial^2 p}{\partial \theta^2} = 0. \tag{3.1}$$

Here, region ABC in the plane (ξ, η) becomes curvilinear triangle ABC in the plane ($x = r_1 \cos\theta$, $y = r_1 \sin\theta$). If we introduce vectors **n**, and s, respectively normal and tangent (in the counterclockwise direction) to the arc AC, the condition for the diffracted front takes the form

$$\frac{\dfrac{\partial p}{\partial n}}{\dfrac{\partial p}{\partial s}} = \frac{Am\operatorname{tg}\theta - B\operatorname{ctg}\theta}{(1 - m^2\sec^2\theta)^{1/2}} = \Omega(\theta). \tag{3.2}$$

Using a series of elementary conformal transformations

$$x_1 = -\frac{(x + ay)}{\sqrt{1 + a^2}},\ y_1 = \frac{(ax - y)}{\sqrt{1 + a^2}},\ z_2 = \frac{\nu - 1}{\mu - 1}\frac{z_1 - \mu}{z_1 - \nu}$$

$$\left(z_1 = x_1 + iy_1,\ \nu = -\frac{\sqrt{1 - M^2} + 1}{M},\ \mu = \frac{\sqrt{1 - M^2} - 1}{M}\right),$$

$$z_3 = z_2^{\pi/\beta}\ (\operatorname{tg}\beta = \sqrt{1 - M^2}\operatorname{tg}\gamma = \frac{\sqrt{1 - M^2}}{a},$$

γ being the angle of reflection of the front Γ_2,

$$z_4 = \frac{z_3 - i}{iz_3 - 1},\ z_5 = -z_4,$$

$$\zeta = \frac{1}{2}\left(z_5 + \frac{1}{z_5}\right) \quad (\zeta = \lambda + i\sigma),$$

we map triangle ABC onto the upper half-plane so that the section AC becomes part of the real axis $-1 < \lambda < 0$ (Fig. 3), section AB becomes part of the real axis $0 < \lambda < 1$, and section CB becomes $|\lambda| > 1$, $\sigma = 0$. In case (a), the point with the coordinates $(-L, 0)$ from the plane (ξ', η') changes into the point

$$\lambda_0 = \frac{2l_1}{1 + l_1^2},\ l_1 = \frac{\nu - 1}{\mu - 1}\frac{l - \mu}{l - \nu},\ l = \frac{\sqrt{1 - L^2} - 1}{L}.$$

In case (b), point G with the coordinates $\left(-\frac{1}{L}, -\frac{\sqrt{L^2 - 1}}{L}\right)$ in the plane (ξ', η') becomes point λ_G on the section $|\lambda| > 1$.

The perturbation problem is then formulated as follows. It is necessary to find a solution to Laplace equations (3.1) in the upper half-plane that satisfies the following conditions:
on the section CA ($\sigma = 0,\ -1 < \lambda < 0$)

$$\frac{\partial p}{\partial \sigma} + \Omega(\theta(\lambda))\frac{\partial p}{\partial \lambda} = 0,$$

on the section AB ($\sigma = 0,\ 0 < \lambda < 1$)

$$\frac{\partial p}{\partial \sigma} = \frac{LM}{\sqrt{1 - L^2}}\delta_0(\lambda - \lambda_0) \quad \text{in case (a)},$$

$$\frac{\partial p}{\partial \sigma} = 0 \quad \text{in case (b)}.$$

on the section CB ($\sigma = 0,\ |\lambda| > 1$)

$$\frac{\partial p}{\partial \lambda} = 0 \quad \text{in case (a)},$$

$$\frac{\partial p}{\partial \lambda} = \frac{LM}{\sqrt{L^2 - 1}}\delta_0(\lambda - \lambda_G) \quad \text{in case (b)}.$$

4. Formulation of a Riemann–Hilbert Problem. Solvability Condition. We introduce the analytic function $\Lambda(\zeta) = p_\sigma + ip_\lambda$ and we formulate a Riemann–Hilbert problem for it. We need to find a function $\Lambda(\zeta) = p_\sigma + ip_\lambda$ which is analytic in the upper half-plane and continuous on $\sigma = 0$, satisfying the following linear relation on the contour

$$q(\lambda)p_\sigma - d(\lambda)p_\lambda = e(\lambda).$$

The functions $q(\lambda)$, $d(\lambda)$ are discontinuous at points A and B. On the section $-1 < \lambda < 0$

$$q = (1 - m^2\sec^2\theta)^{1/2}\mathrm{tg}\,\theta,\ d = (B - Am\mathrm{tg}^2\theta),\ e = 0\ (\theta = \theta(\lambda)),$$

on the section $0 < \lambda < 1$

$$q = 1,\ d = 0,$$

$$e = \begin{cases} \dfrac{LM}{\sqrt{1 - L^2}}\delta_0(\lambda - \lambda_0) & \text{in case (a)}, \\ 0 & \text{in case (b)}. \end{cases}$$

on the section $|\lambda| < 1$

$$q = 0,\ d = 1,$$

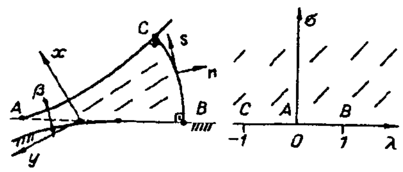

Fig. 3

$$e = \begin{cases} 0 & \text{in case (a)}, \\ -\dfrac{LM}{\sqrt{L^2 - 1}}\delta_0(\lambda - \lambda_G) & \text{in case (b)}, \end{cases}$$

We reduce the Riemann–Hilbert problem to a standard inhomogeneous contact problem on the real axis. Let $\Lambda^+ = p_\sigma + ip_\lambda$, $\zeta \in S^+$ (where S^+ represents the upper half-plane), $\Lambda^-(\zeta) = \bar{\Lambda}^+(\bar{\zeta})$, $\zeta \in S^-$ (S^- being the lower half-plane). We obtain the following contact problem from the conditions of the Riemann–Hilbert problem:

$$\Lambda^+(\lambda) = G(\lambda)\Lambda^-(\lambda) + g(\lambda) \text{ on } \sigma = 0. \tag{4.1}$$

Here, $G(\lambda) = -\dfrac{q - id}{q + id}$; $g(\lambda) = \dfrac{2e}{q + id}$.

In accordance with [3], the following classification of reflection regimes is obtained at $M < 1$

1) $-1 < \Delta_2 < \dfrac{a^2 - R}{a^2 + R}$ a highly stable strong reflected shock;

2) $\dfrac{a^2 - R}{a^2 + R} < \Delta_2 < \dfrac{1 - m^2 - Rm^2}{1 - m^2 + Rm^2}$ a highly stable weak reflected shock;

3) $\dfrac{1 - m^2 - Rm^2}{1 - m^2 + Rm^2} < \Delta_2 < 1 + 2m$.

It is found in calculating the index of problem (4.1) that arg $G(\lambda)$ does not change on the sections $|\lambda| > 1$ and $0 < \lambda < 1$. On the section $-1 < \lambda < 0$ we have

$$3\pi < \Delta \arg G(\lambda) < 4\pi \text{ for regime 1},$$
$$2\pi < \Delta \arg G(\lambda) < 3\pi. \text{ for regime 2},$$
$$0 < \Delta \arg G(\lambda) < \pi \text{ for regime 3}.$$

Choosing the jumps of the argument $G(\lambda)$ at points A and B in accordance with [4] (boundedness is automatic at point C), we obtain the indices for solutions of problem (4.1) that are automatically bounded at point C and integrable (finite) at points A and B, as well as being bounded at infinity ($\varkappa = 3(1)$ for regime 1, $\varkappa = 2(0)$ for regime 2, $\varkappa = 1(-1)$ for regime 3). The pressure boundedness condition means that

$$\Lambda(\zeta) \to 0,\ \zeta \to \infty,\ \zeta\Lambda(\zeta) \to 0,\ \zeta \to \infty. \tag{4.2}$$

To satisfy conditions (2.2), (2.3) and (4.2), we need to choose a solution of problem (4.1) that depends on four real constants. This can be done only when $\varkappa = 3$, which corresponds to regime 1. Regimes 2 and 3 turn out to be indeterminate. In accordance with [4], the solution of the problem is given by the expressions

$$\Lambda(\zeta) = \frac{\Phi(\zeta)}{2\pi i}\int_{-\infty}^{\infty}\frac{\zeta + i}{t + i}\frac{g(t)dt}{\Phi^+(t)(t - \zeta)} + \Phi(\zeta)\left(c_0 + c_1\frac{\zeta - i}{\zeta + i} + c_2\left(\frac{\zeta - i}{\zeta + i}\right)^2 + c_3\left(\frac{\zeta - i}{\zeta + i}\right)^3\right), \tag{4.3}$$

$$\Phi(\zeta) = \begin{cases} \exp \Gamma(\zeta) & \text{at } \zeta \in S^{+}, \\ \left(\frac{\zeta + i}{\zeta - i}\right)^{\varkappa} \exp\Gamma(\zeta) & \text{at } \zeta \in S^{-}, \end{cases}$$

$$\Gamma(\zeta) = \frac{\zeta + i}{2\pi i} \int_{-\infty}^{\infty} \frac{\ln G_0(t)dt}{(t + i)(t - \zeta)},\ G_0(t) = \left(\frac{t + i}{t - i}\right)^{\varkappa} G(t),$$

$$g(t) = \begin{cases} \frac{2LM}{\sqrt{1 - L^2}} \delta_0(t - \lambda_0) & \text{in case (a)}, \\ \frac{2LMi}{\sqrt{L^2 - 1}} \delta_0(t - \lambda_G) & \text{in case (b)}. \end{cases} \quad \varkappa = 3.$$

5. Determination of the Constants c_0, c_1, c_2, c_3. Uniqueness of the Solution. It follows from the representation (4.3) of the solution that

$$p_\lambda(\lambda,\sigma) = \text{Im } F(\zeta) + c_0 \text{Im } \Phi(\zeta) + c_1 \text{Im } \left(\Phi(\zeta) \frac{\zeta - i}{\zeta + i}\right) + c_2 \text{Im} \left(\Phi(\zeta) \left(\frac{\zeta - i}{\zeta + i}\right)^2\right) + c_3 \text{Im} \left(\Phi(\zeta) \left(\frac{\zeta - i}{\zeta + i}\right)^3\right),$$

where

$$F(\zeta) = \frac{\Phi(\zeta)}{2\pi i} \int_{-\infty}^{\infty} \frac{\zeta + i}{t + i} \frac{g(t)dt}{\Phi^{+}(t)(t - \zeta)}.$$

Then conditions (2.2), (2.3) and (4.2) take the form

$$c_0 I_0 + c_1 I_1 + c_2 I_2 + c_3 I_3 = \frac{v_N}{B} - I_4,$$
$$c_0 I_5 + c_1 I_6 + c_2 I_7 + c_3 I_8 = p_N - I_9,$$
$$c_0 + c_1 + c_2 + c_3 = 0,$$
$$3c_0 + c_1 - c_2 - 3c_3 = 0.$$

Here,

$$I_0 = \int_{-1}^{0} \frac{\text{Im } \Phi(\zeta)}{\eta(\lambda)} d\lambda;\ I_1 = \int_{-1}^{0} \frac{\text{Im } \left(\Phi(\zeta) \frac{\zeta - i}{\zeta + i}\right)}{\eta(\lambda)} d\lambda;$$

$$I_2 = \int_{-1}^{0} \frac{\text{Im} \left(\Phi(\zeta) \left(\frac{\zeta - i}{\zeta + i}\right)^2\right)}{\eta(\lambda)} d\lambda;\ I_3 = \int_{-1}^{0} \frac{\text{Im} \left(\Phi(\zeta) \left(\frac{\zeta - i}{\zeta + i}\right)^3\right)}{\eta(\lambda)} d\lambda;\quad I_4 = \int_{-1}^{0} \frac{\text{Im } F(\zeta)}{\eta(\lambda)} d\lambda;\ I_5 = \int_{-1}^{0} \text{Im } \Phi(\zeta) d\lambda;$$

$$I_6 = \int_{-1}^{0} \text{Im} \left(\Phi(\zeta) \frac{\zeta - i}{\zeta + i}\right) d\lambda;\ I_7 = \int_{-1}^{0} \text{Im} \left(\Phi(\zeta) \left(\frac{\zeta - i}{\zeta + i}\right)^2\right) d\lambda;\quad I_8 = \int_{-1}^{0} \text{Im} \left(\Phi(\zeta) \left(\frac{\zeta - i}{\zeta + i}\right)^3\right) d\lambda;\ I_9 = \int_{-1}^{0} \text{Im } F(\zeta) d\lambda.$$

The constants c_0, c_1, c_2, and c_3 are determined as

$$J = \det \begin{vmatrix} I_0 & I_1 & I_2 & I_3 \\ I_5 & I_6 & I_7 & I_8 \\ 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & -3 \end{vmatrix} \neq 0.$$

We will show that this condition is satisfied. Let the above determinant be equal to zero and let f_1 and f_2 be nontrivial solutions of problem (4.1), (2.2), (2.3), (4.2). We will examine $f = f_1 - F_2$. For the pressure p corresponding to solution f, conditions (2.2) and (2.3) take the form

$$\int_{AC} p_\lambda d\lambda = 0; \tag{5.1}$$

$$\int_{AC} \frac{p_\lambda d\lambda}{\eta(\lambda)} = 0. \tag{5.2}$$

Conditions (4.2) remain unchanged. By virtue of the Zarembo–Gero lemma, the minimum and maximum of pressure p cannot be reached on section AB (see Fig. 2) because $\partial p/\partial n = 0$, on section BC because $\partial p/\partial s = 0$, or on section AC because of condition (3.2). The only exception is point D, where $\theta = 0$. Inside the region, $\Delta p = 0$. Thus, extrema can be reached only at points A, B, C, and D. If a minimum is reached at point A, then in accordance with condition (5.1) $p_A = p_B = p_C$. A maximum is reached only at point D ($p_D = p_*$). Integrating condition (5.2) by parts, we obtain

$$\frac{p - p_*}{\eta(\lambda)}\Big|_A - \frac{p - p_*}{\eta(\lambda)}\Big|_C + \int_C^A \frac{p - p_*}{\eta^2} d\eta = 0$$

Here, all three terms are of the same sign, so that $p - p_* = 0$. Thus, min p = max p, which means that p = const and f = 0. Then contact problem (4.1) is unambiguously solvable within the chosen class of functions and $J \neq 0$.

The solvability of the above-formulated boundary-value problem therefore depends on the type of reflected shock wave. If the latter is weak and highly stable, there will be no change in the character of the flow, i.e., the reflected shock will be distorted and the perturbation problem will have a solution. The given self-similar perturbation problem cannot be solved in other cases, which indicates that the flow undergoes a fundamental restructuring.

An analysis of solution (4.3) reveals that it has a logarithmic singularity at the point of inflection. This shows the inappropriateness of using a linear approximation in a small neighborhood of the point and the need for a separate detailed study of the behavior of the solution near this point. A full investigation of this matter is beyond the scope of the present study.

The authors of [3] reached similar conclusions in a study of nonsteady non-self-similar perturbations of a steady flow created with the regular reflection of an oblique shock from a rigid wall. An analysis of the equations of the transonic approximation led the authors of [5] to also conclude that the perturbation problem could not be solved for a strong shock wave.

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